### **Dimensionality Reduction**

**Question 1**: Note: In this question, all columns will be written in their transposed form, as rows, to make the typography simpler. Matrix M has three rows and three columns, and the columns form an orthonormal basis. One of the columns is [2/7, 3/7, 6/7] and another is [6/7, 2/7, -3/7]. Let the third column be [x, y, z]. Since the length of the vector [x, y, z] must be 1, there is a constraint that x2+y2+z2 = 1. However, there are other constraints, and these other constraints can be used to deduce facts about the ratios among x, y, and z. Compute these ratios.

Given Matrix M =

If columns in a matrix form an orthonormal base, then:

* The dot product between any two columns is Zero.

Then, C1.C3 🡺 0

* 🡪 equation 1
* 🡪 equation 2

Rewriting the above equations, we get

1 🡺

2 🡺

Eq.1/6 🡺 🡪 equation 3

Eq.2/3 🡺 🡪 equation 4

Eq.3 + Eq.4 🡺

Eq.1 x 3 🡺 🡪 equation 5

Eq.5 – Eq.2 🡺

Therefore

**Question 2**: Find the eigenvalues and eigenvectors of the following matrix:



You should assume the first component of an eigenvector is 1. Then, find out One eigenvalue and One eigenvector.

Given Vector A =

We know that 🡪 🡪equation 1

A - = - =

= (2 - (10 - ) – 9

* 20 - 12 + – 9

Hence the Eigen values 🡪

**For:**

* = 0

**For:**

**Question 3**: Suppose [1,3,4,5,7] is an eigenvector of some matrix. What is the unit eigenvector in the same direction? Find out the components of the unit eigenvector.

Unit Eigen Vector in the same direction

**Question 4**: Suppose we have three points in a two dimensional space: (1,1), (2,2), and (3,4). We want to perform PCA on these points, so we construct a 2-by-2 matrix, call it N, whose eigenvectors are the directions that best represent these three points. Construct the matrix N and identify, its elements.

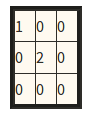
Given elements in two dimensional space are (1, 1), (2, 2) and (3, 4)

* M =

Transpose of =

Therefore

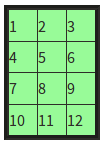
**Question 5**: Consider the diagonal matrix M =



Compute its Moore-Penrose pseudoinverse.

Moore-Penrose pseudoinverse

**Question 6**: When we perform a CUR decomposition of a matrix, we select rows and columns by using a particular probability distribution for the rows and another for the columns. Here is a matrix that we wish to decompose:



Calculate the probability distribution for the rows.

Probability distribution of the rows:

The sum of squares of elements of A is 1+4+9+16+25+35+49+64+81+100+121+144 = 650

Squared frobenius norm for all rows and their probabilities are:

**Row 1:**

Probability =

**Row 2:**

Probability =

**Row 3:**

Probability =

**Row 4:**

Probability =

Probability distribution of rows is 🡪 0.021, 0.118, 0.298, 0.561